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Numerical modeling of continuation problems
for the acoustic and Helmholtz equation

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 - Mathematical and computer modeling"

Relevance of the topic. The dissertation work is devoted to the development of a scientific direction that is relevant for applications - the development and study of numerical simulations for solving the continuation problem for the acoustics and Helmholtz equations. The search for causes while recognizing their consequences is the idea of solving inverse problems of wave equations. From a theoretical point of view, the calculation of the sound field of a point or distributed source is reduced to solving a boundary value problem for the Helmholtz wave equation in inhomogeneous regions. The study of a wave experiment in natural conditions is costly, especially when studying the fields of sound waves in the ocean. Due to this, a numerical experiment is especially important, which is set up under controlled conditions with full repeatability of input and output data, with any practically necessary accuracy, in any options for changing the parameters of the problem.

Geophysical methods play an important role in studying the structure of the internal structure of the Earth. They are based on measurements of the characteristics of a certain physical field on the earth's surface, which carries information about the structure of the Earth. Such fields, in particular, are acoustic and electromagnetic fields, which in the case of acoustics depend on the wave propagation velocity and density, and in electrodynamics, on the conductivity of magnetic permeability. The solution of the continuation problem on a timelike surface for hyperbolic equations was studied by M.M.Lavrentiev, V.G.Romanov, R.Courant, S.P. Shishatsky. Later V.G.Romanov obtained a priori estimates for the solution of the Cauchy problem with data on a timelike surface. The main approach to solving inverse problems, including problems of geophysics, is based on the preliminary solution of direct problems in a selected class of models. Further determination of the parameters of the medium is reduced to an approximate description of the experimental field by solving the direct problem. At the same time, according to the idea of V.N.Strakhov, it is possible to solve the inverse problem without solving the direct one, having only a differential operator, to which the field depends. In many cases, gravity measurement data do not allow us to determine the location of drilling exploration wells unambiguously. Therefore, it is advisable to pre-calculate the anomalous gravitational field at a certain depth under the Earth's surface using the available data. In this connection, there is a significant practical demand for the development and analysis of new mathematical models of geophysical processes, which allow solving both direct and inverse problems with higher efficiency. Inverse problems in gravity exploration were solved in most cases by the selection method and only in the two-dimensional case. The issues of uniqueness and stability of solutions of inverse geophysical problems have been studied to a limited extent, but nevertheless certain significant results in this direction have been obtained, including the P. S. Novikov’s theorem uniqueness of the solution. Thus, the problem of developing accurate, efficient and high-speed algorithms for solving direct and inverse problems of geology and other fields of science in various formulations is relevant. In connection with the development of the field of study of geophysics and other listed sciences, it became necessary to create a science of methods for solving the inverse problem. In the works of M.M.Lavrentiev, V.G.Romanov, S.I.Kabanikhin, A.Lorenzi, A.M.Denisov, M.Klibanov, M.Grasseli and others studied linear and nonlinear inverse problems for equations of hyperbolic, parabolic type in various formulations. Such problems are characterized by some properties that are unpleasant from the point of view of computer technology - non-uniqueness, instability with respect to the error of the initial data, and require special algorithms developed on the basis of strong theories based on the use of the capabilities of modern computing technologies. Understanding which model and method to use in a particular situation requires some knowledge of both the model approximation error and the method discretization error. In this regard, at present, much attention is paid to the development of efficient methods and algorithms for solving various classes of inverse problems. At present, new formulations of inverse problems and, consequently, new results on their solvability are continuously emerging.

Recent applications of the continuation problem of the acoustic equations are found in the field of noise control and soundproofing. This reduces noise and pollution in environments such as urban areas, workplaces and homes. An urgent problem is the development and development of efficient numerical methods for solving inverse problems for such applications. It is time to propose approaches and methods for creating a unified theory of inverse problems encountered in various fields of applications.

The aim of the dissertation work is to create and investigate the solvability of the inverse continuation problem for the acoustic and Helmholtz equation. Construction of high-speed and economical iterative methods for the considered problems. Creating a program for the implementation of the considered numerical methods. Ensuring an increase in the accuracy, speed and stability of solving direct and inverse problems.

Research objectives. To achieve this goal, it was necessary to solve the tasks:

1. Study and development of methods for solving the continuation problem for the acoustics equation

2. Creating an algorithm for the numerical solution of the inverse problem for one-dimensional and two-dimensional acoustic equations based on the method of gradients, the method of inversion of finite-difference schemes.

3. To investigate and develop numerical methods for solving the ill-posed initial-boundary value problem for the Helmholtz equation.

4. Numerical study of the stability of direct and inverse problems for the Helmholtz equation.

5. Development of programs for implementation of the considered methods.

The object of research is the algorithm for constructing a numerical solution of the continuation problem for the acoustics and Helmholtz equations.

Research methods. For the numerical solution of the models under consideration, use methods that allow solving the problem of boundary reconstruction. Landweber method, gradient method, finite difference method, and method inverse difference schemes were used. The programming languages C++, Python were used for numerical modeling.

The scientific novelty of the work is as follows:

- An effective numerical algorithm based on the method of inversion finite-difference scheme for the boundary inverse problem of the acoustic equation in a triangular domain was constructed.
- Using the directional derivative, the gradient of the objective functional of the boundary inverse problem for the acoustics equation in the time-like triangular prism domain is found. An algorithm for solving the inverse problem is constructed. The novelty of the problem is that the two-dimensional problem is considered in the area of a triangular prism.
- An efficient numerical algorithm based on the Landweber method for solving the inverse problem of recovering two unknown boundary conditions for the Helmholtz equation in a quadrilateral region is developed and numerical results are obtained.

Provisions made for the defense. According to the results of the study, the following provisions are defended:

- Numerical solution of the boundary inverse problem using the method of inversion of difference schemes. Computational experiments with different noise levels have been implemented.
- Reduction of an ill-posed problem into an inverse problem, in the triangular prism domain, and calculation the gradient of the functional.
- Application of the projection method for the direct problem in solving a two-dimensional inverse problem of acoustics in a timelike triangular domain, as well as the convergence of the iterative Landweber method.
- An optimization method for solving the initial-boundary value problem for the Helmholtz equation, in which depth data are used along with surface data, shows that adding new data will allow a more stable solution.
- A numerical study of the stability of the initial-boundary value problem for the Helmholtz equation shows the instability of the problem, since the condition number tends to infinity.

Theoretical and practical significance of the research. The results obtained in the dissertation are new and have a scientific character. The presented results are of theoretical and practical value. The paper considers the development and justification of numerical methods for solving acoustic problems as the most important scientific direction for its application. The theoretical and practical significance of the dissertation work lies in the developed numerical method for solving the considered direct and inverse problems. The use of algorithms and programs developed in the dissertation significantly increases the efficiency of solving the problems under consideration. In the future, the results of the work can be applied in the study and development of the theory of inverse problems arising in wave fields. The developed algorithms and programs have the practical possibility of being used in prospecting seismology and acoustic tomography, as well as, according to recent studies, in the study of noise isolation.

Work approbation. The main research results were reported and discussed at the following conferences:

- International Conference: «Inverse problems in finance, economics and life». (Almaty, December 26-28, 2017).

- Computational and Information Technologies in Science, Engineering and Education. №998, p.197-207. https://www.springer.com/series/7899 (Revised Selected Papers CITech-2018, Ust-Kamenogorsk, September 25-28, 2018).

- International April Mathematical Conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 1150th anniversary of Abu Nasyr al-Farabi (Almaty, April 1-3, 2020).

- 6th International Conference of Mathematical Sciences (ICMS 2022), (Maltepe University, Istanbul, Turkey, July 20-24, 2022.)

- Proceeding of the 8th International Conference on Control and Optimization with Industrial Applications, Volume 1, (Baku, Azerbaijan, August 24-26, 2022).

- International Conference «Computational and Information Technologies in Science, Engineering and Education» (CITech-2022) dedicated to the 90th anniversary of Academician N. K. Nadirov, to the 80th anniversary of Academician M. O. Otelbaev. (Almaty, October 12-15, 2022).

Publications. The results of the dissertation work were published in 12 papers. Including 2 publications in ranked journals, 4 articles in journals recommended by committee for quality assurance of education and science of the RK, 6 theses in the proceedings of international conferences.

Structure and scope of work. The dissertation consists of an introduction, three chapters, a conclusion, a list of references. The total volume of work is 103 pages. The list of references contains 95 titles.

Brief summary of the work. The introduction provides a review of publications related to the topic of the dissertation, substantiates the relevance of the research topic. The purpose of the work, scientific novelty and practical significance of the results are formulated, the content of the work is briefly summarized.

The first chapter is devoted to the construction of original economical algorithms for solving the continuation problem for the one-dimensional acoustics equation. The one-dimensional problem of continuation from a part of the boundary of the solution of the acoustics equation is studied. The idea of the continuation problem is to find the value of the desired function in the rest of the boundary using additional information in a certain part of the boundary. The original problem is reduced to an inverse boundary value problem. A finite-difference scheme is constructed for the inverse problem under consideration, and an unknown function is found from this difference equation by inverting the difference scheme. To demonstrate the effectiveness and simplicity of the method under consideration, a benchmark problem is given. Numerical experiments and graphs of these numerical results are given.

The second chapter of the dissertation is devoted to the formulation of the two-dimensional inverse problem of acoustics. The goal of this chapter is to construct an efficient algorithm for the numerical solution of a two-dimensional inverse problem for the acoustics equation from data on a time-like surface. The main result of the chapter is the application of the projection method for the direct problem in solving the two-dimensional inverse problem of acoustics in a time triangular domain. An algorithm for solving the inverse problem for the acoustics equation is constructed. Numerical results are presented.

The third chapter is devoted to the development of numerical solutions of direct and inverse problems for the Helmholtz equation. If the law of oscillation of the physical medium harmonically depends on time, then the wave equation can be transformed into the Helmholtz equation. The main result of the chapter is to obtain a formula for calculating the gradient of the functional through the solution of the direct and conjugate problems and to construct an algorithm for solving the continuation problem. The singular values of the operator A for the original and direct problems are presented and analyzed. Also at the end are presented the results of numerical experiments showing the effectiveness of the method.

In conclusion, the main results on the topic of the dissertation are given, the prospects for applying the results obtained and the prospects for further work in the chosen direction are discussed.

The conclusion contains the main results on the topic of the dissertation and discussed the prospects for the application of the results and the prospects for further work in the chosen direction.